Review of Potential function & stream function

Stream function: ψ

i) Derived with Continuity

ii) valid for 2-D flow

iii) For incompressible flow, the difference between two streamlines is the volume flow rate between those streamlines

Potential function: ϕ

i) Exist only for irrotational flow.

ii) valid for 3-Dimensional flow

iii) Is everywhere $\perp$ to ψ

iv) Find ϕ from u & v same as ψ

In fluid flow if the flow is incompressible and inviscid (irrotational)

Ideal flow

Continuity equation $\quad \nabla \cdot \vec{V} = 0$

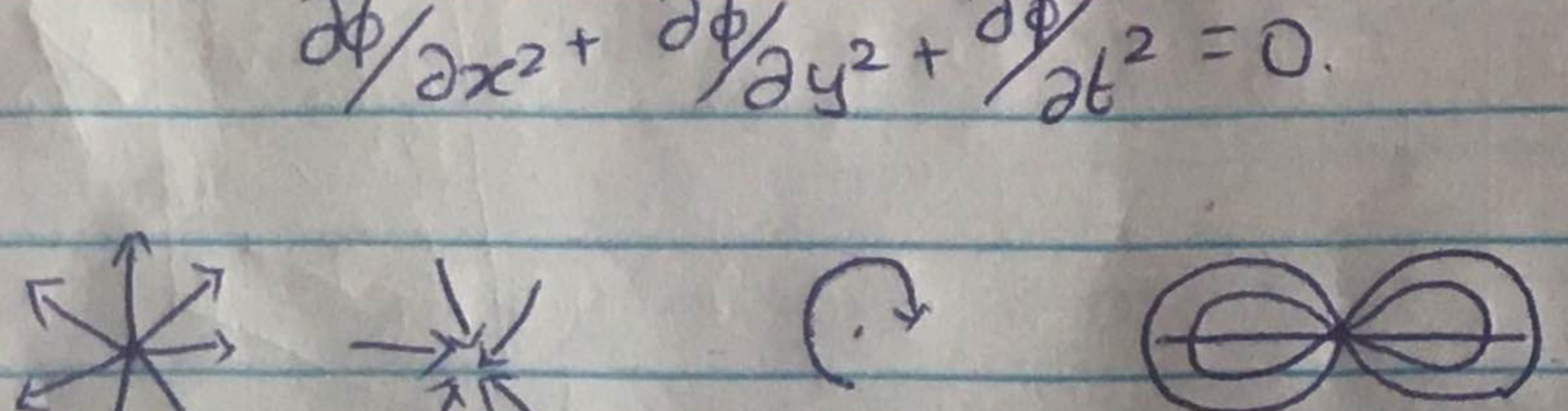
IRRotational $\quad \nabla \times \vec{V} = 0$

$\therefore \vec{V} = \nabla \cdot \phi \rightarrow$ Potential function

$$\nabla \cdot \vec{V} = \nabla \cdot (\nabla \phi) = \nabla^2 \phi = 0.$$

$\nabla^2 \phi = 0 \quad$ Laplace equation

$$\partial^2\phi/\partial x^2 + \partial^2\phi/\partial y^2 + \partial^2\phi/\partial t^2 = 0.$$



Combining the fluid flow & Identifying the properties

Rotational & Irrotational flow.

$\vec{\omega}$ = Angular velocity vector.

$$\vec{\omega} = \omega_x i + \omega_y j + \omega_z k$$

$$\vec{r} = xi + yj + k.z$$

$$\vec{V} = \vec{\omega} \times \vec{r}$$

$$= \begin{vmatrix} i & j & k \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix}$$

$$ui + vj + wk = i\underbrace{(z\omega_y - y\omega_z)}_{u} + j\underbrace{(x\omega_z - z\omega_x)}_{v} + k\underbrace{(y\omega_x - x\omega_y)}_{w}$$

curl $\nabla \times \vec{V}$

$$= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ (z\omega_y - y\omega_z) & (x\omega_z - z\omega_x) & (y\omega_x - x\omega_y) \end{vmatrix}$$

$$= i(\omega_x + \omega_x) + j(\omega_y + \omega_y) + k(\omega_z + \omega_z)$$

$$\nabla \times \vec{V} = 2[\omega_x i + \omega_y j + \omega_z k]$$

$$\nabla \times \vec{V} = 2\vec{\omega}, \qquad \vec{\omega} = \frac{1}{2} \nabla \times \vec{V}.$$

vorticity $= 2\vec{\omega} = \nabla \times \vec{V}$.

$\nabla \times \vec{V} = 0$ irrotational flow.

$\nabla \times \vec{V} \neq 0$ rotational flow.